

MODELLING AND IDENTIFICATION OF A LABORATORY HELICOPTER

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Abstract. In this paper modelling and identification of a laboratory helicopter with the final aim to design a control system is presented. The CE 150 made by Humusoft is a laboratory helicopter designed for studying system dynamics and control engineering principles. First, the helicopter set-up is depicted. Next, theoretical and empirical modelling is systematically described. Identification of the necessary parameters is tackled and the results are presented. Finally, model validation is discussed and a simple control approach is proposed.

1. Introduction

Modelling and simulation are very important approaches for designing control systems. Therefore, laboratory set-ups, which model real processes, and mathematical models have a significant role [1–3]. The CE150 is a laboratory helicopter made by Humusoft [4]. It is used for studying system dynamics and control engineering principles from the theoretical point of view and enables a wide range of practical experiments in the fields of modelling, simulation and control. The goal of modelling and identification is to prepare a basis for the students' laboratory assignments, such as designing a multivariable controller that ensures satisfactory control in the whole operating range. There are two well known modelling approaches: theoretical and experimental. Usually, both approaches have to be combined, which is also the case in modelling of the laboratory helicopter.

2. The laboratory helicopter set-up

The laboratory helicopter set-up (see figure 1) comprises a helicopter body carrying two motors, which drive the main and the tail propeller of the helicopter, and a servomechanism, which shifts the centre of gravity by moving a weight along the helicopter's horizontal axis. The helicopter body is connected to a base so that two degrees of freedom are enabled:

- rotation around the horizontal axis \rightarrow pitch angle ψ ;
- rotation around the vertical axis \rightarrow azimuth φ .

The axes of the main and tail rotor and the vertical and horizontal helicopter axis are perpendicular to each other. The helicopter model can be represented as a non-linear multi-variable system with three inputs (measured in machine units $[-1, 1]$):

- u_1 – voltage driving the main motor;
- u_2 – voltage driving the tail motor;
- u_3 – position of the servomechanism (weight)¹;

and two outputs (measured in radians):

- ψ – pitch angle;
- φ – azimuth.

¹ From now on, let us presume that the weight never moves from the neutral position during operation of the helicopter ($u_3 = 0$). Otherwise, we should model the movement as a disturbance that affects both the centre of gravity and the moment of inertia of the helicopter body. However, this is beyond the scope of this paper.

The inputs u_1 and u_2 are measured in machine units ranging from -1 to 1 . An interface unit, which connects the helicopter and the computer, converts the inputs from machine units to appropriate voltage values that drive the motors.

Output ψ denotes the pitch angle, i.e. the angle between the vertical axis and the longitudinal axis of the helicopter body, whereas φ denotes the azimuth, i.e. the angle in the horizontal plane between the longitudinal axis of the helicopter body and its zero position. Both angles are measured in radians.

The voltage driving the main motor u_1 and the voltage driving the tail motor u_2 affect both the pitch angle ψ and the azimuth φ , therefore we can say that the mentioned interactions make the system multivariable. However, it is possible to fix one (or both) degree of freedom by tightening the intended screw(s) in the helicopter base when needed.

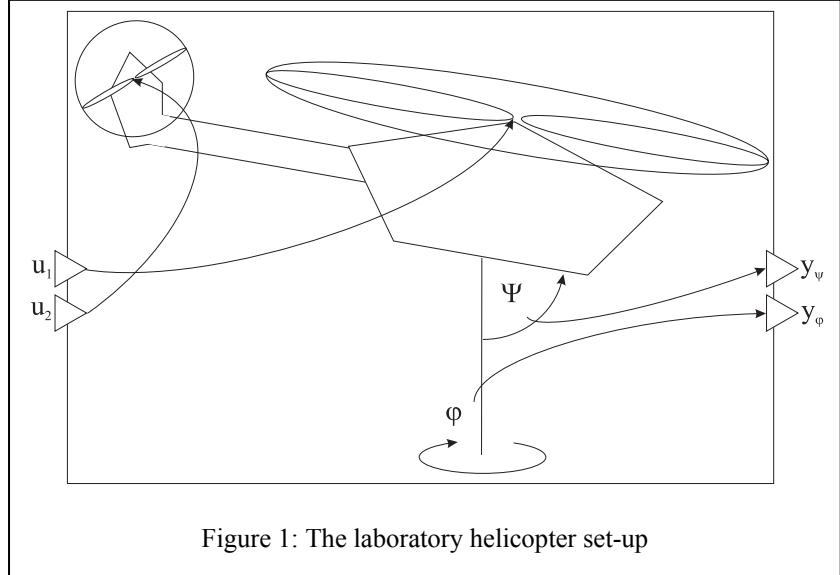


Figure 1: The laboratory helicopter set-up

There are analogue connections between the helicopter and the interface unit, which converts the signals from analogue to digital and vice versa. The interface unit is connected to a computer via a multifunction input/output card. All the experiments are done in Matlab-Simulink environment using Real Time Toolbox [5].

3. Theoretical modelling

When modelling a system it is important to find a balance between simplicity and complexity of the model, according to its purpose and operating conditions. The model has to be clear, concise and flexible, yet it must consider all the relevant sub-processes in the system.

Modelling of dynamic systems is a cyclic process, therefore usually many iterations are needed before a satisfactory model is obtained [6]. Sometimes validation of a particular sub-system gives unsatisfactory results. Hence, another approach has to be considered and some of the previously neglected properties have to be taken into account. In the following section, modelling procedure will be described. Obviously, only the last iteration of the procedure is presented in the paper and the model is proposed in its final form.

3.1. Modelling of sensors

Both angle sensors are linear, so the modelling is rather straightforward.

$$\Psi = k_{\psi} \cdot y_{\psi} + y_{\psi 0} \quad (1)$$

$$\varphi = k_{\varphi} \cdot y_{\varphi} + y_{\varphi 0} \quad (2)$$

Signals from the sensors are denoted by y_{ψ} and y_{φ} .

3.2. Vertical plane dynamics

We start with torque balance equation around the horizontal axis.

$$J_1 \ddot{\Psi} = \tau_1 - \tau_{F_g} - \tau_{B\Psi} - \tau_G \quad (3)$$

Here, J_1 denotes the moment of inertia around the horizontal axis, τ_1 the torque of the propulsion force of the main propeller, τ_{F_g} the torque of the gravitation force of the helicopter body, $\tau_{B\Psi}$ the friction torque and τ_G the gyroscopic effect caused by rotation of the main propeller and rotation of the helicopter body around the vertical axis.

The torque of the propulsion force of the main motor is modelled experimentally. The static characteristic is derived from the dominant ventilator characteristic as described in eq. (4). The motor-propeller dynamics are relatively fast comparing to the dynamics of the helicopter body. Therefore, it can be modelled as a 2nd order transfer function with two equal poles – see eq. (5).

$$\tau_1 = a_1 \cdot u_{1mot}^2 + b_1 \cdot u_{1mot} \quad (4)$$

$$\frac{U_{1mot}(s)}{U_1(s)} = \frac{1}{(T_1 \cdot s + 1)^2} \quad (5)$$

The other torques are defined using basic physical laws.

$$\tau_{F_g} = F_g \cdot l \cdot \sin \Psi = M_g \cdot \sin \Psi \quad (6)$$

$$\tau_{B\Psi} = B_{\psi 1} \cdot \dot{\Psi} + B_{\psi 2} \cdot \text{sign}(\dot{\Psi}) \quad (7)$$

$$\tau_G = K_{gyro} \cdot \tau_1 \cdot \dot{\varphi} \cdot \cos \Psi \quad (8)$$

In eq. (6), F_g is the gravitation force, l is the lever between the centre of gravity and the horizontal axis of the helicopter body, and M_g is the appropriate torque. In eq. (7) we assume that the friction torque is a sum of a Coulomb part and a linear part, where the latter is proportional to the angular velocity. The torque caused by the gyroscopic effect (see eq. (8)) is proportional to the product of the angular velocity of the main propeller, the angular velocity of the body around the vertical axis, and the cosine of the pitch angle Ψ . It is presumed that τ_1 is proportional to the angular velocity of the main motor. The constant of the gyroscopic coupling is denoted by K_{gyro} .

3.3. Horizontal plane dynamics

We start with torque balance equation around the vertical axis.

$$J_2 \ddot{\varphi} = \tau_2 - \tau_{B\varphi} - \tau_r \quad (9)$$

Here, J_2 denotes the moment of inertia around the vertical axis, τ_2 the torque of the propulsion force of the tail propeller, $\tau_{B\varphi}$ the friction torque, and τ_r the reaction torque caused by the main propeller rotation.

Eqs. (10), (11) and (12) are defined in the same manner as in the previous subsection.

The other torques are defined using basic physical laws.

$$\tau_2 = a_2 \cdot u_{2mot}^2 + b_2 \cdot u_{2mot} \quad (10)$$

$$\frac{U_{2mot}(s)}{U_2(s)} = \frac{1}{(T_2 \cdot s + 1)^2} \quad (11)$$

$$\tau_{B\varphi} = B_{\varphi 1} \cdot \dot{\varphi} + B_{\varphi 2} \cdot \text{sign}(\dot{\varphi}) \quad (12)$$

The reaction torque caused by the main propeller rotation is also modelled experimentally. The static characteristic is derived from the dominant ventilator characteristic as described in eq. (13). The dynamics are modelled as a transfer function in eq. (14) where the denominator is the same as in eq. (5). Due to the moment of inertia of the main rotor, which affects the reaction torque, numerator dynamics are assumed as well.

$$\tau_r = a_r \cdot u_{1r}^2 + b_r \cdot u_{1r} \quad (13)$$

$$\frac{U_{1r}(s)}{U_1(s)} = \frac{T_{1r} \cdot s^2 + T_{2r} \cdot s + 1}{(T_1 \cdot s + 1)^2} \quad (14)$$

3.4. The whole system – Simulink model

Using eqs. (1) – (14) a Simulink block diagram of the whole system depicted in figure 2 was developed.

4. Measurements and identification of the parameters

Once the theoretical model of the laboratory helicopter set-up is obtained, 22 parameters have to be determined: $k_\psi, y_{\psi 0}, k_\varphi, y_{\varphi 0}, M_g, J_1, B_{\psi 1}, B_{\psi 2}, J_2, B_{\varphi 1}, B_{\varphi 2}, T_1, a_1, b_1, T_2, a_2, b_2, T_{1r}, T_{2r}, a_r, b_r$ in K_{gyro} . There are two possible approaches:

- direct measurements of the accessible physical quantities;
- identification, i.e. experimental estimation of the parameters by means of measuring inputs and outputs [7–9].

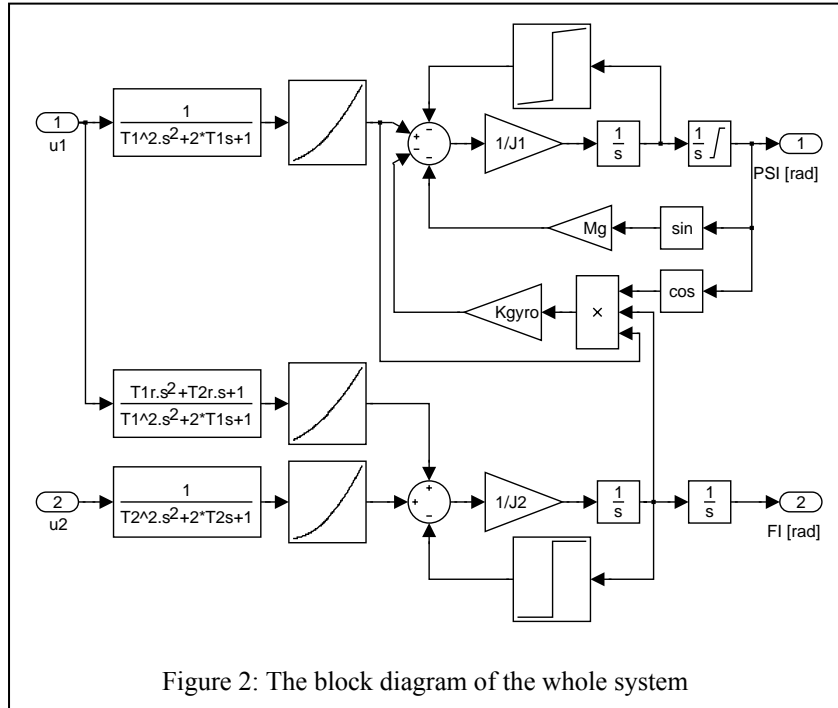


Figure 2: The block diagram of the whole system

The angle sensors: $k_\psi, y_{\psi 0}, k_\varphi, y_{\varphi 0}$

The sensors can be calibrated by simple angle measurements.

$$k_\psi = \pi / 1024 \text{ [rad]},$$

$$y_{\psi 0} = 275 \pi / 1024 \text{ [rad]},$$

$$k_\varphi = \pi / 1024 \text{ [rad]},$$

$$y_{\varphi 0} = 0.7 \pi \text{ [rad]}.$$

The torque of the gravitation force of the helicopter body: M_g

The parameter M_g is estimated by hanging N weights weighing m_i ($i = 1, \dots, N$) on the helicopter body behind and in front of the horizontal axis. We can obtain M_g by measuring the pitch angle ψ and the appropriate levers l_i in steady state.

$$M_g = \frac{\sum_{i=1}^N m_i \cdot g \cdot l_i}{\sin \Psi} \quad (15)$$

In order to mitigate the effect of Coulomb friction on the measurements and thus obtain better accuracy, it is reasonable to carry out many measurements with different weights hung at different places and then calculate the average M_g .

$$M_g = 0.07088 \text{ N m}$$

Vertical plane dynamics: $J_1, B_{\psi 1}, B_{\psi 2}$

The parameters concerning the vertical plane dynamics are estimated by means of identification. When conducting this experiment both motors are turned off and rotation in the horizontal plane is disabled by tightening the corresponding screw. Due to constrained motion of the pitch angle ψ , the model base is fixed in the *perpendicular position*, which means that the helicopter base is tilted 90 degrees so that the vertical axis is put in a horizontal position. Obviously, the new position has to be considered when calibrating the sensor (offset $\pi/4$). The helicopter response to an initial condition (tilt of the helicopter body) is finally recorded.

The parameters are identified by offline model adaptation, which is a method suitable for nonlinear identification problems such as the laboratory helicopter. In this case, the minimisation of the criterion function, which takes into account the sum of squares of the difference between the real response y_{real} and the model response y_{mod} to the initial condition (see eq. (16)), is carried out using the Nelder-Mead method.

$$\min_{J_1, B_{\psi 1}, B_{\psi 2}} \sum_{k=1}^N (y_{real}(k, \mathbf{u}) - y_{mod}(k, \mathbf{u}, J_1, B_{\psi 1}, B_{\psi 2}))^2 \quad (16)$$

Before the optimisation it is reasonable to roughly estimate the parameters from the response signal, in order not to end up in a local minimum that provides inadequate results. The optimisation returns the following parameters.

$$\begin{aligned} J_1 &= 2.43 \cdot 10^{-3} \text{ kg m}^2 \\ B_{\psi 1} &= 1.53 \cdot 10^{-4} \text{ kg m}^2 \text{ s}^{-1} \\ B_{\psi 2} &= 1.03 \cdot 10^{-4} \text{ kg m}^2 \text{ s}^{-1} \end{aligned}$$

Horizontal plane dynamics: $J_2, B_{\phi 1}, B_{\phi 2}$

The parameters concerning the horizontal plane dynamics are identified in a similar fashion. Both motors are turned off and rotation in the vertical plane is disabled by tightening the corresponding screw. The model base is fixed in the *perpendicular position*, therefore the influence of the torque of the gravitation force of the helicopter body has to be considered in the adjusted model as well. The helicopter response to an initial condition (tilt of the helicopter body) is finally recorded. Again, the parameters are identified by offline model adaptation.

$$\begin{aligned} J_2 &= 2.02 \cdot 10^{-3} \text{ kg m}^2 \\ B_{\phi 1} &= 5.50 \cdot 10^{-6} \text{ kg m}^2 \text{ s}^{-1} \\ B_{\phi 2} &= 5.96 \cdot 10^{-4} \text{ kg m}^2 \text{ s}^{-1} \end{aligned}$$

The torque of the propulsion force of the main propeller: T_1, a_1, b_1

The parameters a_1 and b_1 represent the static characteristic of the main motor-propeller subsystem. They can be identified from the measurements of the static characteristic.

$$a_1 \cdot u_1^2 + b_1 \cdot u_1 = M_g \sin \psi \quad (17)$$

Using the least squares method, the following parameters are obtained.

$$a_1 = 0.1244 \text{ N m}$$

$$b_1 = 0.0496 \text{ N m}$$

Parameter T_1 is identified by offline model adaptation to a step response signal of the helicopter body.

$$T_1 = 0.0904 \text{ s}$$

The torque of the propulsion force of the tail propeller: T_2, a_2, b_2

In a similar manner as in the previous subsection, but with the helicopter in the *perpendicular position*, the following parameters are obtained.

$$a_2 = 0.1959 \text{ N m}$$

$$b_2 = 0.0202 \text{ N m}$$

$$T_2 = 0.0567 \text{ s}$$

The reaction torque caused by the main propeller rotation: T_{1r}, T_{2r}, a_r, b_r

In a similar manner as in the previous subsection, again with the helicopter base in the *perpendicular position*, the following parameters are obtained.

$$a_r = 0.0148 \text{ N m}$$

$$b_r = 0.0108 \text{ N m}$$

$$T_{1r} = 0.0017 \text{ s}$$

$$T_{2r} = 0.1908 \text{ s}$$

The gyroscopic effect:

K_{gyro}

Parameter K_{gyro} is identified by offline model adaptation. The main motor input u_1 is set to a constant value, so that it can be assumed the angular velocity of the main propeller is constant. In this manner, the system should stabilize at a certain pitch angle ψ , which should not be too close to $\pi/4$. Next, the body of the helicopter is rotated² around its vertical axis, so that the angular velocity is approximately constant. The rotation causes the gyroscopic effect, which results in a change of the pitch angle. The signals ψ and φ are recorded. The angular velocity around the vertical axis is derived ($d\varphi / dt$) and used for identification as a subsystem input.

$$K_{gyro} = 0.3185 \text{ s}$$

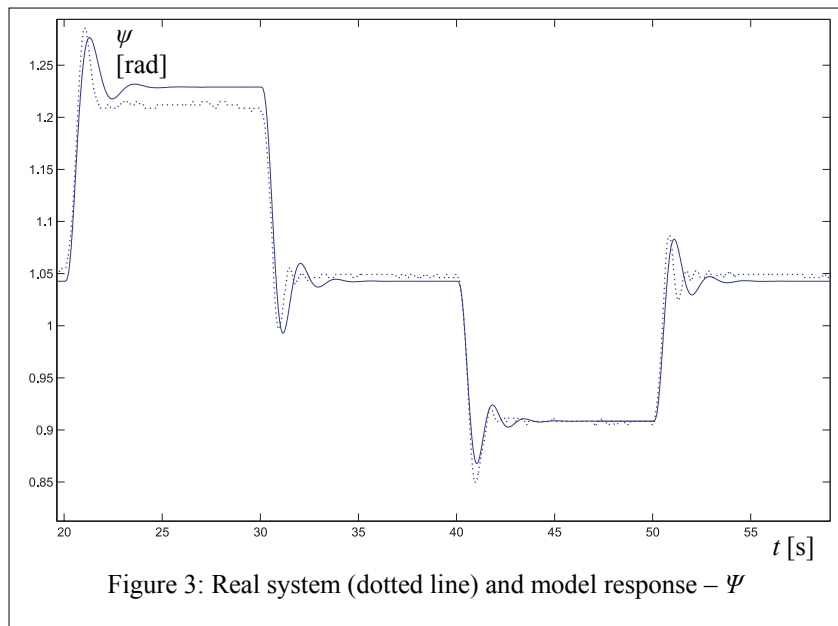


Figure 3: Real system (dotted line) and model response – ψ

² By hand, without touching the body.

5. Model validation

Model validation is carried out in several steps. First, the dynamics in the vertical plane are validated. Since the process is operating in a stable region, an open loop experiment is conducted. In this case, input u_1 is a multi-step signal ($u_1 = 0,53 \rightarrow 0,56 \rightarrow 0,53 \rightarrow 0,50 \rightarrow 0,53$). The comparison between the response of the pitch angle ψ of the real helicopter (dotted line) and of the model (solid line) is shown in figure 3.

Due to instability, the horizontal plane dynamics can not be validated in an open loop experiment. Therefore, a controller³ has to be provided, which will enable the tracking of an azimuth reference. The azimuth reference φ_{ref} is a multi-step signal ($\varphi_{ref} = 0 \rightarrow \pi/4 \rightarrow -\pi/4 \rightarrow 0$). The comparison between the response of the azimuth φ of the real helicopter (dotted line) and of the model (solid line) is shown in figure 4. In both cases the same controller is used. In the case of the real system response, we can see that the azimuth φ is subject to significant external disturbances.

Figure 5 shows the input signal u_2 provided by the controllers in case of the real system⁴ (dotted line) and in case of the model (solid line). From the comparison we can see that the signals are quite similar in the transient states. However, there are some perceivable differences, especially in the interval from 125 s to 155 s, which occur due to the torque of the Coulomb friction.

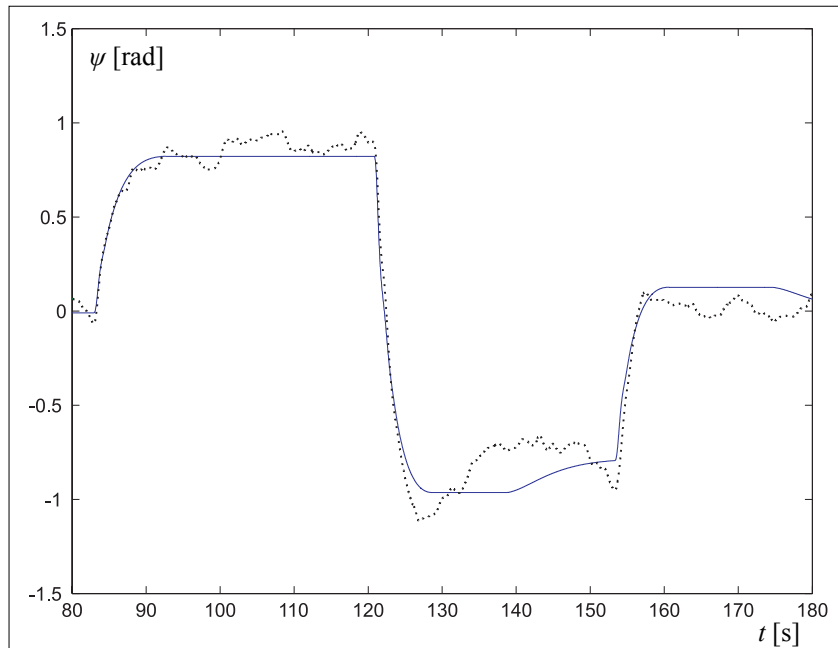


Figure 4: Real system (dotted line) and model response – φ

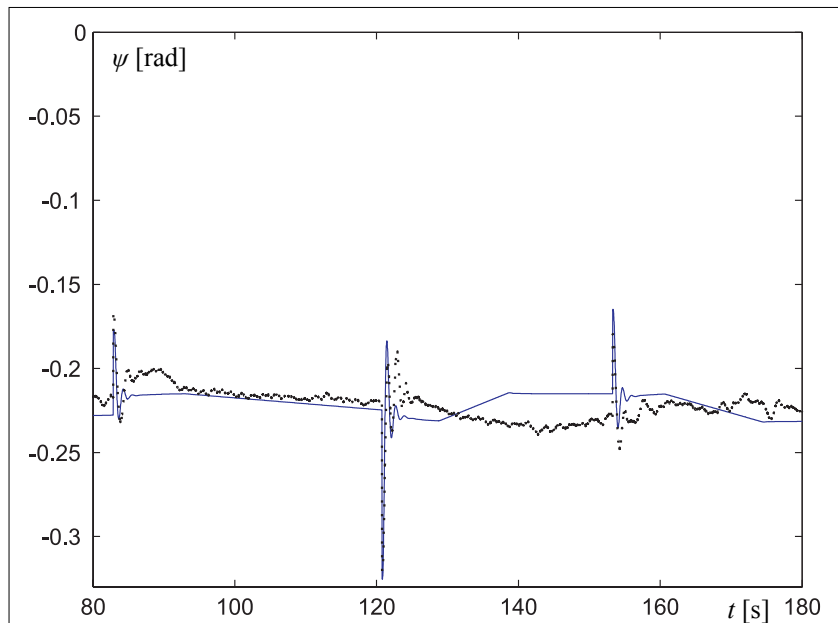


Figure 5: Real system (dotted line) and model response – u_2

The Coulomb friction causes the helicopter body to stop rotating not only if the input u_2 is set to a certain exact value, but also if it is set in a narrow band around that value. This means that two slightly different input signals

³ A modified PI-D controller is used.

⁴ The input signal u_2 is the average of 20 measurements. This way it is possible to reduce the impact of noise and the effect of drift of the parameters. There is especially noticeable drift of the parameters in the static characteristic of the tail motor-propeller subsystem, therefore it sometimes needs adjustment, in our case by factor $k = 0.88$.

u_2 can cause the same output signal φ . Obviously, the maximum torque difference generated by the two input signals is $2 \cdot B_{\varphi 2}$. Eq. (18) proves that the input signal difference causes a torque difference that is smaller than the mentioned bound.

$$\left| \frac{d\tau_2}{du_2} \right|_{u_{2,0} \approx -0,22} \cdot \Delta u_2 \Big|_{\max} = \left| (2 \cdot a_2 \cdot k^2 \cdot |u_{2,0}| + b_2 \cdot k) \cdot \Delta u_{2\max} \right| \approx \underline{\underline{0,9 \cdot 10^{-3} Nm \leq 1,2 \cdot 10^{-3} Nm = 2 \cdot B_{\varphi 2}}} \quad (18)$$

When assessing the usability of the model, we encountered a simulation problem; namely, because of the non-linearity caused by Coulomb friction, the simulation runs very slowly, despite the use of the solver for stiff systems (*ode23s*). Hence, it is reasonable to replace the nonlinear function with a piecewise linear function, i.e. to introduce a very high gain (in our case 10^8) and an appropriate saturation as a substitute for infinite gain around zero angular velocity. Such modification has practically no influence on the results; however, it does speed up the simulation considerably and what is more, even enables real-time⁵ experiments and thus allows for online comparison of real and simulated signals.

6. Control

As mentioned, the developed mathematical model is intended for the design of control systems. In this section, a simple control approach is presented. Two independent modified PI-D controllers for each degree of freedom respectively are used, without considering the multivariable nature of the system. However, such basic approach can still turn out useful for stabilizing the helicopter body in a reference position, as shown in figure 6.

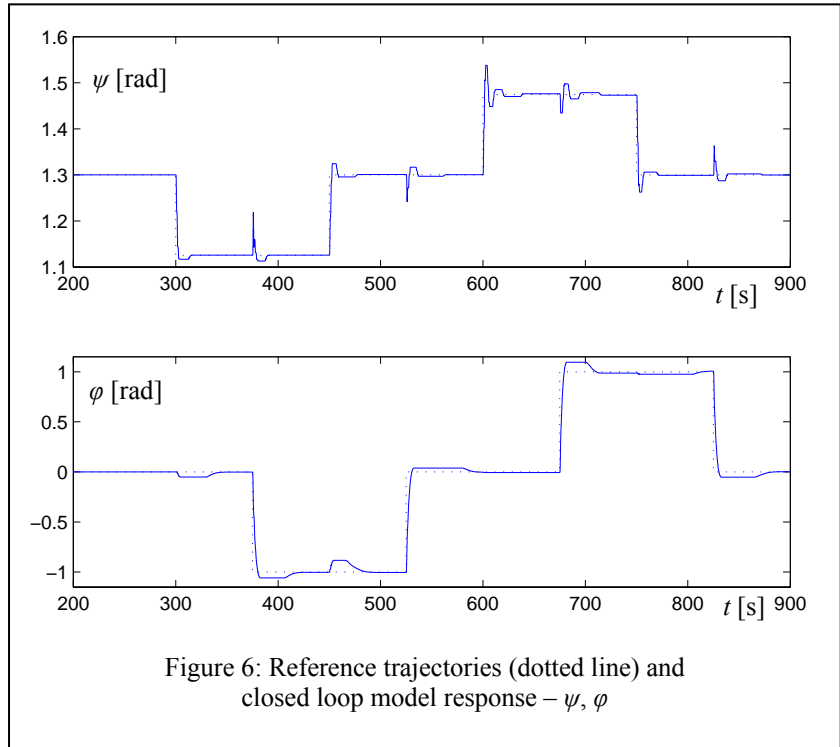


Figure 6: Reference trajectories (dotted line) and closed loop model response – ψ, φ

7. Conclusions

In the paper, modelling and identification of a laboratory helicopter was dealt with. The CE 150 laboratory helicopter made by Humusoft was presented as a multivariable system with two inputs and two outputs. Next, modelling of the helicopter was systematically tackled by disassembling the system into simpler subsystems, i.e. modelling of the sensors, vertical plane dynamics and horizontal plane dynamics. Furthermore, the vertical and horizontal plane dynamics have been analysed by modelling each relevant torque separately. In addition, measurement and identification of all the parameters needed was illustrated. Finally, validation of the developed mathematical model was treated. The validation results suggest that the developed mathematical model adequately represents the laboratory helicopter. In conclusion, a simple control approach was presented, which involves two independent modified PI-D controllers for each degree of freedom respectively.

⁵ Matlab 7.0.1 on Celeron 2.4 GHz, 512 MB.

8. References

- [1] Cellier, F. E., Continuous System Modelling, Springer Verlag, New York, 1991.
- [2] Egeland, O. and Gravdahl, J. T., Modeling and Simulation for Automatic Control, Marine Cybernetics, Trondheim, 2002.
- [3] Karba, R., Modeliranje procesov, Založba FE in FRI, Ljubljana, 1999.
- [4] Humusoft, CE 150 helicopter model: User's manual, Humusoft, Prague, 2002.
- [5] Humusoft, Real time toolbox for use with SIMULINK: User's manual, Humusoft, Prague, 2002.
- [6] Matko, D., Zupančič, B. and Karba, R., Simulation and modelling of continuous systems: a case study approach. Prentice Hall international series in systems and control engineering, Prentice Hall, New York, 1992.
- [7] Juang, J. N., Applied System Identification, Prentice Hall, Englewood Cliffs, 1994.
- [8] Ljung, L., System Identification: Theory for the user, Prentice Hall, Englewood Cliffs, 1987.
- [9] Lebon, F., Contact problems with friction: models and simulation. Simulation modelling practice and theory, 5-6 (2003), 449 – 463.
- [10] Matko, D., Identifikacije, Založba FE in FRI, Ljubljana, 1998.